Behavior of Concrete Deep Beams Reinforced with Inclined Web Reinforcement around Different Opening Shapes

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ABSTRACT

The objective of this paper is to experimentally and analytically estimate the influence of inclined reinforcement placed above and below web openings having different shapes in reinforced concrete (RC) deep beam. Twenty RC deep beams had the same overall geometric dimensions were tested under two-point top loading. Test variables included amount of inclined reinforcement, opening shape (circular, square, rectangular and relatively new type rectangular with fillet edges) and shear span-to-depth ratio. The relationship between the amount of inclined reinforcement and the opening size was expressed as the effective inclined reinforcement factor. As this factor was increased, the behavior of tested beams improved, where the crack width and its development decreased, and the ultimate load increased. The improvement rate of ultimate load with increasing effective inclined reinforcement for beams with rectangular openings having fillet edges was higher than that with sharp edges. Beams with opening having square, circular, or rectangular with fillet edges shapes and having effective inclined reinforcement ratio above 0.085 and 0.091 under shear span-to-depth ratio 1.0 and 0.6, respectively had higher ultimate load than that of corresponding solid beams. The effect of inclined reinforcement on enhancing the behavior of deep beam with opening increased as the shear span-to-depth ratio decreased. The ultimate load of tested beams was estimated using upper-bound analysis of the plasticity theory and compared with the test results. It is shown that the prediction has a consistent agreement with the experimental results.

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1. INTRODUCTION

Deep beams are defined in many codes [1-4], as a structural element having a ratio of clear span to effective depth is equal to four or lower, or a ratio of shear span to effective depth is equal to two or lower. Creating openings in the shear zone area of RC deep beams is frequently required to pass the utility services. Numerous studies have been conducted on the behavior of RC deep beams with openings [5-14]. The openings reduce the shear capacity of RC deep beams and this reduction depend on the degree of interruption of the opening to the load path from the load point to the support point. Placing web reinforcement around openings improves the shear capacity of deep beams, and the enhancement in beam strength is greater when inclined reinforcement is used than when vertical and horizontal reinforcement are used [14-16]. Yang, et al. [10] investigated the relation between the amount of inclined reinforcement and opening size. Test results showed that the shear strength of tested beams improved significantly depending on the amount of inclined reinforcement relative to the opening size.

In this research, twenty RC deep beams were tested under two-point top loading. The variables considered were the opening shapes (circular, square, rectangular and rectangular with fillet edges), shear span-to-depth ratio and amount of inclined web reinforcement around openings. The effect of the amount of inclined reinforcement on the structural behavior of tested beams combined with the opening size in a factor named the effective inclined reinforcement factor [10] and can be calculated as presented in this paper.

The classic plane-sections remain plane hypothesis cannot apply to deep beams, which is considered as discontinuity regions and the strain distribution is significantly nonlinear. The plasticity theory is the most commonly used for predicting the shear strength of deep beam [17]. For rigid structures, the plasticity theory mainly includes the lower-bound theorem and upper-bound theorem. Design of deep beams in many codes [1-4] is allowed by strut-and-tie model which relies on the lower-bound theorem. The upper-bound theorem is more capable to predict the shear capacity of deep beams with web openings. The ultimate load capacity for tested beam were compared with that estimated using the upper-bound analysis of the plasticity theory.

1.1 Effective Inclined Reinforcement Factor ($\xi$)

The failure plan in deep beam with openings is formed from upper and lower diagonal cracks connecting the opening corners and the edges of load plates of load and support points. The tension force in concrete at the failure plane is resisted by the component force of inclined reinforcement perpendicular to the failure plane, as shown in Fig. 1.

![Fig. 1. Identification of symbols in effective inclined reinforcement factor](image)

The total load capacity produced by inclined reinforcement placed above and below the opening crossing the failure planes ($\Sigma F_{DT}$) can be expressed as follows

$$
\Sigma F_{DT} = n_t (A_s f_y) \sin (\beta + \Theta_v) + n_b (A_s f_y) b \sin (\beta_b + \Theta_b) 
$$

Where:

- $A_s$, $f_y$, $n$: Area, yield strength and number of inclined reinforcement, respectively.
- $\beta$: The angle between inclined reinforcement and the beam longitudinal axis; and
- $\Theta_v$: The angle between the opening line and the vertical axis.
2. EXPERIMENTAL PROGRAM

Twenty deep beams were cast and tested to investigate the shear behavior of RC deep beams with web openings. The main variables were the opening shapes, the amount of inclined reinforcement around openings and the shear span-to-depth ratio (a/h). The web openings of all tested beams had the same height of 120 mm and positioned at the center of load path between applied load point and supporting point. The openings used were circular, square, rectangular, and relatively new type rectangular with fillet edges. The dimensions of openings are shown in Fig. 2. For each opening shape, three beams were tested, one without web reinforcement around opening and the other two beams with different amount of inclined reinforcement, which placed in layers above and below the openings, each consisting of 2010. The number of inclined reinforcement layers varied from one to three to give effective inclined reinforcement factor, calculated using Eq. (4), from 0.045 to 0.165. The inclination of web reinforcement to the longitudinal axis of beams was chosen 45 degrees. Table 2 shows the details of tested beams.

The nomenclature for tested beams is shown in Fig. 3, where the first letter indicates the presence of inclined reinforcement around opening, (U) for beams without inclined reinforcement and (I) for beams with inclined reinforcement. The second letter points to the opening shape, where (C) for circular shape, (S) for square, (R) for rectangular with sharp edges and (F) for rectangular with fillet edges. For solid beam, the second letter was (N). Shear span-to-depth ratio denoted with the third letter, 10 for (a/h) equal 0.1 and 6 for (a/h) equal 0.6. Finally, the number after the dash refers to the number of inclined steel bars in the top chord above the openings or in the bottom chord below the openings.

All tested beams had cross section (100X400) mm and 1200 mm long with an effective span 1000 mm having span-to-depth ratio of 2.5. The shear span was 400 mm and 240 mm to give a shear span-to-depth ratio of 1.0 and 0.6, respectively. The tested beams were designed to induce shear failure before any flexural distress. The beams were reinforced with 4Ø12 in the tension side and 2Ø8 in the compression side. The tension and compression reinforcement were continuous over the whole length of the beam and had 90° hook at their ends to provide sufficient anchorage. The web reinforcement was Ø 6 @ 150 mm spacing in both vertical and horizontal directions. Additional horizontal web reinforcement added over the supporting points.

\[ \Theta: \text{The angle between failure plane and the beam longitudinal axis.} \]

Subscripts \( t \) & \( b \) refer to the top chord above openings and bottom chord below opening, respectively.

The total transverse tensile force in concrete developed across failure plane can be expressed as follows

\[ \sigma_T b (h_o/sin\theta_b) + (h - D - h_o)/sin\theta_t \]  \hspace{1cm} (2)

Where:

- \( b \), \( h \): The width and height of the beam, respectively.
- \( w \), \( D \): The width and height of the opening, respectively.
- \( \sigma_T \): The transverse tensile stress in concrete at failure planes; and
- \( h_o \): The distance between the beam soffit and the bottom face of the opening.

The ratio of \( \sigma_T \) to \( f_y \), where Tan, et al. [9] and Kong et al. [13,14] mentioned that the bottom chord is more critical than the top chord, can be obtained from Eq. (3) as given in the following

\[ \frac{\sigma_T}{f_y} = \frac{n_t(A_t)(f_T)(f_y)l_1(\beta + \theta_l) + n_p(A_p)l_2(f_y)l_1(\beta + \theta_p)}{b(t/o)l_2(\sin(\theta_b) - \sin(\theta_t))} \]  \hspace{1cm} (3)

The right-hand side of the previous equation indicates the effective inclined reinforcement ratio \( \rho_{aot} \) which provides sufficient resistance to the transverse tensile stresses in the concrete. The increase of the opening area led to reduction in the section area and cause higher transverse tensile stresses. The opening area ratio \( \rho_{aot} \) represents the ratio of the opening area, \( A_o \), to the shear span area, \( a.h \). The effective inclined reinforcement factor \( \xi \) is the ratio of effective inclined reinforcement ratio \( \rho_{aot} \) to the opening area ratio \( \rho_{aot} \) and can be expressed as follows

\[ \xi = \frac{\rho_{aot}}{a_h} = \frac{n_t(A_t)(f_T)(f_y)l_1(\beta + \theta_l) + n_p(A_p)l_2(f_y)l_1(\beta + \theta_p)}{b(h_o/sin\theta_b + (h - D - h_o)/sin\theta_t)} \]  \hspace{1cm} (4)

2.1 Specimens and Text Matrix

Twenty deep beams were cast and tested to investigate the shear behavior of RC deep beams with web openings. The main variables were the opening shapes, the amount of inclined reinforcement around openings and the shear span-to-depth ratio (a/h). The web openings of all tested beams had the same height of 120 mm and positioned at the center of load path between applied load point and supporting point. The openings used were circular, square, rectangular, and relatively new type rectangular with fillet edges. The dimensions of openings are shown in Fig. 2. For each opening shape, three beams were tested, one without web reinforcement around opening and the other two beams with different amount of inclined reinforcement, which placed in layers above and below the openings, each consisting of 2010. The number of inclined reinforcement layers varied from one to three to give effective inclined reinforcement factor, calculated using Eq. (4), from 0.045 to 0.165. The inclination of web reinforcement to the longitudinal axis of beams was chosen 45 degrees. Table 2 shows the details of tested beams.
to avoid local failure. Diagonal-shaped inclined reinforcement was placed in top and bottom chords of the openings. The inclined reinforcement had sufficient anchorage length, according to ACI 318-08 to prevent anchorage failure and slippage at diagonal crack planes. A clear cover of 15 mm and 10 mm were kept at the top and bottom of the beam and the vertical sides of the beam, respectively. A schematic of tested beams showing the dimensions and reinforcement details is shown in Fig. 4.

2.2 Material Properties

The material used in concrete mixture for tested beams were Ordinary Portland Cement (OPC-42.5 grade), natural sand with 2.6 fineness moduli and 16 mm maximum aggregate size crushed dolomite. The target 28 days concrete compressive strength ($f_{cu}$) was 35 MPa. The actual $f_{cu}$ was determined from cubes casted and cured simultaneously with the tested beams. The steel bars of 6 mm and 8 mm diameter were mild steel, and steel bars of 10 mm and 12 mm diameter were high grade steel. The measured yield strength of 10 mm and 12 mm diameter were 450 and 530 MPa, respectively.

2.3 Test Setup and Instrumentation

The beams were tested to failure under four-points bending. The beams were placed in 1000 KN capacity rigid reaction frame, and loaded using 1000 KN capacity hydraulic jack connecting to an electric pump. The load was transferred to the tested beam using a spreader steel beam. At the load and supporting points, a steel plates having 30 mm thickness and 100 mm width were...
Table 1. Details of tested beams

<table>
<thead>
<tr>
<th>Beam code</th>
<th>(a/h) ratio</th>
<th>Shape and size of opening</th>
<th>Inclined reinforcement</th>
</tr>
</thead>
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<td></td>
<td></td>
<td>Shape</td>
<td>$A_o$</td>
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<td>1600</td>
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</tr>
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<td>Circular</td>
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<td>0.071</td>
</tr>
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<td>0.071</td>
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<td>Square</td>
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<td>0.090</td>
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<td>Fillet$^1$</td>
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</tr>
<tr>
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<td>Fillet$^1$</td>
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</tr>
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<td>Fillet$^1$</td>
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<td>0.118</td>
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<td>144.0</td>
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<tr>
<td>IS6-4</td>
<td>Square</td>
<td>144.0</td>
<td>0.150</td>
</tr>
</tbody>
</table>

1 Rectangular with fillet edges; 2 Shear span area; 3 Ratio of opening area to shear span area; 4 Effective inclined reinforcement ratio estimated using Eq. (3); and 5 Effective inclined reinforcement factor estimated using Eq. (4)

Fig. 5. Test setup for tested beams

placed to prevent bearing failure. Three Linear Variables Differential Transformer (LVDT) mounted at the bottom face of the beam at the mid span and directly under the two loading points to monitor the vertical deflections. The side surfaces of the beams were painted in a white color to aid on the observation of cracks development during testing. The crack width at concrete strut joining the edge of support plate and opening corner opposite to the support point was monitored by a strain gage. All test data were captured using data acquisition and recorded using at intervals of two seconds. Fig. 5 shows the details of test setup.
3. EXPERIMENTAL RESULTS AND DISCUSSION

3.1 Load and Mid-Span Deflection

Fig. 6 shows the applied load versus the mid-span deflection for tested beams. The initial stiffness of all tested beams, until the inclined cracks occurred, was nearly the same. After the inclined cracks formed, the deflection sharply increased for beams having no inclined reinforcement. Inclined reinforcement around opening greatly affected on the stiffness of the beam and its effect increased as the numbers of reinforced bars increased. The stiffness of beams having four or six inclined reinforced bars in the top chord over the openings and in the bottom chord below the openings were higher than that of solid beams. Which means that increasing the numbers of inclined reinforced bars around web openings in deep beams could be overcame the deterioration stiffness with the existence of the opening. The effect of inclined reinforcement on the beam stiffness increased as shear span-to-depth ratio decreased.

3.2 Diagonal Crack Width

Fig. 7 shows the relationship between applied load and the development of diagonal crack width formed between the edge of support plate and opening corner opposite to the support point. For solid beams, the crack width was measured on the load path connecting the edges of support and load plates. For beams without inclined reinforcement, the crack width greatly increased after the occurrence of first diagonal crack. This behavior, however, difficulty occurred for beams having inclined reinforcement. The cracking width and its development rate decreased with the increase in the effective incline reinforcement ratio. Moreover, the shear span –to- depth ratio affected the cracking behavior of beams. Reducing the shear span–to-depth ratio decreased the crack width and its development rate.

![Graphs showing load versus mid-span deflection for different opening types and shear span-to-depth ratios](image)

**Fig. 6 (a-b). Load verses mid-span deflection curves**

**Fig. 7 (a-b). Diagonal crack width for different opening types and shear span-to-depth ratios**
Table 2 summarized the test results. The first cracking load \( (P_{cr}) \) greatly dropped with the existence of the opening in the shear zone area of tested beams. The decreasing in \( P_{cr} \) was slightly effected by the opening shape. Installing inclined reinforcement around opening enhanced \( P_{cr} \) in the range from 25-83%. \( P_{cr} \) for solid beam and beams with openings increased with the decrease in the shear span-to-depth ratio. The ultimate load \( (P_u) \) for deep beams having web openings and without inclined reinforcement was dramatically decreased compared with that of the solid beam. In Fig. 8, the ultimate load for beams without inclined reinforcement under the shear span-to-depth ratio of 1.0and 0.6, respectively, were plotted against the ratio of opening area to shear zone area \( (\rho_{oa}) \). In this plot, the opening with fillet edges enhanced the beam capacity. Where, the ultimate load for Beam UF10 having a rectangular opening with fillet edges and \( \rho_{oa} = 0.121 \) reduced only 6.5% of that for beam UC10 with a circular opening and \( \rho_{oa} = 0.071 \), and increased 21% of that for Beam UR10 with a rectangular opening having sharp edges and \( \rho_{oa} = 0.135 \). It is confirmed that the fillet edges of openings redistributed the tensile stress consideration around openings better than the sharp edges. In addition, from the plot, the ultimate load for deep beam having openings and without inclined reinforcement increased with reducing the shear span-to-depth ratio.

The inclined reinforcement around opening greatly improved the ultimate capacity of deep beam. Fig. 9 shows the variation of ultimate load of beams against the effective inclined reinforcement factor \( (\xi) \) according to each variable. The ultimate load for all tested beams was directly proportional to the effective inclined reinforcement factor with different rate according to openings shapes and shear span-to-depth ratio. The gain in \( P_u \) with the increasing \( \xi \) was larger for rectangular openings with fillet edges than that with sharp edges. The effect of the effective inclined reinforcement factor on ultimate load significantly enhanced with reducing the

3.3 Diagonal Cracking and Ultimate Load

Fig. 7 (a-b). Load versus diagonal crack width
shear span–to-depth ratio. Deep beam with circular, square or rectangular with fillet edges and having an effective inclined reinforcement factor of more than 0.085 and 0.091 under shear span–to-depth ratio 1.0 and 0.6, respectively, exhibited higher ultimate load than the corresponding solid beams.

3.4 Cracking Behavior

From the observation while the experimental work was carried out and from the crack’s configuration of tested beams, all the deep beams where failed in shear. Fig. 10 shows examples of 10 types of tested deep beams after failure. For solid deep beams, the flexural cracks were initiated at the beams bottom about 33% of maximum load, and these are followed by diagonal cracks at both sides of deep beams. With load increasing the flexural cracks stopped and more inclined cracks appeared in the compression strut joining the load and support points. The first cracks for deep beams having opening formed near the top and the bottom

<table>
<thead>
<tr>
<th>Beam code</th>
<th>$f_{cu}$ (Mpa)</th>
<th>Cracking Load $P_{cr}$ (KN)</th>
<th>Ultimate Load $P_u$ (KN)</th>
<th>$(P_u)_{pre.}$ (KN)</th>
<th>$(P_u)<em>{exp}$ $(P_u)</em>{pre}$</th>
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</table>

Fig. 8. Relation between $\rho_{oa}$ and $P_u$ for beams tested without inclined reinforcement
Fig. 9. Relation between $\xi$ and $P_u$

(i) Opening Shape  
(ii) $(a/h)$ ratio

Fig. 10. Crack patterns at failure
of opening corners in the range of 21-31% and 12-21% of maximum load for beams without inclined reinforcement and with inclined reinforcement, respectively. These cracks propagated toward the support and load points with increasing applied load and followed by bottom flexural cracks at the mid-span of the beam, and then diagonal cracks created at opening corners opposite to support and load points. The development of diagonal cracks increases with increasing the effective inclined reinforcement factor.

All tested beams except the solid beams failed along the inclined cracks joining the opening corners and the edge of load or support plates. Failure planes divided the deep beams into two blocks, which had relative transitional and rotational displacements between them. The observed failure plane of deep beams with opening suggest the use of upper-bound solution of the plasticity theory to predict the ultimate load for these beams as presented in the following paragraph.

4. THEORETICAL PREDICTION FOR LOAD CAPACITY

Many investigator [10,11,18,19] proposed the upper-bound theorem of the plasticity theory to predict the shear capacity of reinforced concrete beams with openings. In this approach, the failure mode, as presented in Fig. 11, consisted of two yield lines (or failure zones) separate the beam to two rigid blocks. The rigid block I relative rotate about instantaneous center (IC) having coordinates (Xic and Yic) as shown in Fig. 11. The upper and lower yield lines are separated by the opening and having different angle and displacement rate about IC.

The energy due to the relative rotation of rigid block I is dissipated on both the concrete along the yield lines and the inclined reinforcing bars crossing the yield lines. The position of IC produces a minimum shear strength (upper-bound approach) is the solution of model. The plasticity theory is based on equating the internal energy estimated from the internal stresses in both concrete (Wc) and reinforcing bars (Ws) along the yield lines and external work done (Wa) calculated from the displacement rate of the support reaction in rigid block I (δ) relative to the center of the upper and lower yield lines, as in the following equations:

\[ W_c = \frac{1}{2} b_w \omega [(f'c_s r_i (1 - \sin \alpha_s) l_i) + (f'c_s r_b (1 - \sin \alpha_b) l_b)] \quad (5) \]

\[ W_s = \sum_{i=1}^{n} \omega (A_s i) (f')_i (r_i) \cos (\Psi_i)_i \quad (6) \]

\[ W_k = (P_k/2)_{pre} \omega |x_{ic}| \quad (7) \]

From \( W_k = W_c + W_s \)

\[ (P_k)_{pre} = \frac{2h}{|x_{ic}|} (f'c_s r_b (1 - \sin \alpha_s) \frac{h}{\sin \theta_b} + f'c_s r_i (1 - \sin \alpha_b) \frac{h - d - h_u}{\sin \theta_i} + 2 \sum_{i=1}^{n} \frac{A_s i}{b_i} (f')_i (r_i) \cos (\Psi_i)_i ) \quad (8) \]

Where:

- \( r_i \): the distance between the midpoint of the chord of the yield line and the instantaneous center
- \( \omega \): the relative rotational displacement of rigid block II to rigid block I about IC
- \( \alpha \): the angle between the relative displacement at midpoints of the chord and yield line
- \( \psi_s \): the angle between the relative displacement about IC and the reinforcing bar \( i \) crossing the yield line
- \( l \): the length of the yield line
- \( f'_c \): effective compressive strength = \( v_a f'_c \)

![Fig. 11. Failure mode of deep beam with web opening](image-url)
Following conclusions may be drawn.

- Diagonal cracking width of the tested beams, the carrying capacity, load deflection behavior and base the effective inclined reinforcement factor ($\xi$).
- The amount of inclined reinforcement with the load path between the load and the support plates. The openings center were located at the level of the longitudinal bottom reinforcement. The horizontal coordinate of the instantaneous center was iteratively tuned until achieving the minimum load capacity for beam.
- The estimated values of ultimate load for tested beams using the upper-bound plasticity model were compared with the experimented results. As shown in Table 2. The present mechanism analysis slightly under estimates the shear capacity. The mean and standard deviation of the ratio ($P_{exp}/P_{cal}$) were 0.945 and 0.13 respectively.

5. CONCLUSION

Twenty RC deep beams with web openings were tested under two-point top loading. The test parameters included the opening shape, the amount of inclined reinforcement placed at the top chord above the opening and the bottom chord below the opening and the shear span-to-depth ratio. The openings center were located at the load path between the load and the support points. The amount of inclined reinforcement with respect to the opening size was expressed as the effective inclined reinforcement factor ($\xi$). Based on the analysis and compared load carrying capacity, load deflection behavior and diagonal cracking width of the tested beams, the following conclusions may be drawn.

- The initial stiffness of beams was not affect by the amount of inclined reinforcement. After the occurrence of diagonal cracks, the deflection significantly increased for beams without inclined reinforcement. The beam stiffness increased with the increase in the amount of inclined reinforcement and the decrease in the shear span-to-depth ratio. The beam stiffness furthermore, increased more than the solid beams for large amount of the inclined reinforcement.
- All tested beams failed in shear with the expected diagonal failure plane joining the opening corners and the opposite edge of the load and support plates.
- Diagonal crack width and its development rate decreased with the increase in the effective inclined reinforcement factor and the reduction in the shear span-to-depth ratio.
- Fillet edges for rectangular openings without inclined reinforcement affected on increase of the ultimate load for beams than the sharp edges. Also, the ultimate load for beams having no inclined reinforcement increased with the decrease in shear span-to-depth ratio.
- The ultimate load is directly proportional with the increase in the effective inclined reinforcement factor for all tested beams. The effect of effective inclined reinforcement factor on increasing ultimate load was higher for beams with rectangular openings having fillet edges than that having sharp edges. Beams having opening with square, circular or rectangular with fillet edges shapes, the ultimate load capacity was higher than the corresponding solid beams when effective inclined reinforcement factor increased than 0.085 and 0.091 under shear span-to-depth ratio 1.0 and 0.6, respectively.
- The reduction of shear span-to-depth-ratio significantly improved the effect of inclined reinforcement factor on the load capacity of tested beams.
- The prediction of the ultimate capacity of the deep beam with opening using the upper-bound analysis of the plasticity theory was very reasonable and higher with an average 5.5% compared with experimental results.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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